Complex Geometry Exercises

Week 5

Exercise 1. *Show that the following hypersurfaces are smooth and compute their Kodaira dimension:*

(a) $Z(x_0^2 + z_1^2 + z_2^2) \subset \mathbb{CP}^2$

(b) $Z(x_0^3 + z_1^3 + z_2^3) \subset \mathbb{CP}^2$

(c) $Z(x_0^5 + \cdots + z_5^5) \subset \mathbb{CP}^4$

Exercise 2. *Show that the image of* $\mathrm{Div}(X) \to \mathrm{Pic}(X)$ *is given by the classes of line bundles which admit a non-zero meromorphic section.*

Exercise 3 (Bezout's theorem)**.** *Let* $C, D \subset \mathbb{P}^2$ *smooth (distinct) curves defined by homogeneous polynomials* f *and* g *of degrees* d *and* e *respectively.*

(i) *Show that the line bundle* $\mathcal{O}(1)$ *restricted to* C *is of degree* d*.*

(ii) *Show that*

$$d \cdot e = \sum_{p \in C \cap D} \dim_{\mathbb{C}} \mathcal{O}_{\mathbb{P}^2,g}/(f, g).$$

Exercise 4. *Let* X *be a smooth complex manifold with* $K_X \cong \mathcal{O}_X$*. Show that* X *cannot be obtained by a blow-up of another surface.*

Exercise 5. *Let* X *be a smooth Calabi-Yau manifold (with* $K_X \cong \mathcal{O}_X$*). Show that* X *cannot be obtained by a blow-up of another surface.*